Part II: Nonlinear stability theory

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Abstract

Section 5: Introduction
In part I of the course, we considered an initial base state subject to perturbations of small amplitude. We distinguished the case of linear instability, in which the small perturbations grow in time, from that of linear stability, in which they decay. Unfortunately, linear stability analysis is unable to tell us what happens for perturbations of finite amplitude, for which nonlinear effects are important. In this part of the course, therefore, we turn our attention to nonlinear dynamics.

Section 6: A review of linear stability analysis
Before doing so, however, we pause to review what we know about linear stability analysis from part I of the course. We also undertake a critique of it, in order to motivate in more detail the need to study nonlinear effects.

Section 7: Local bifurcation theory
A key concept in this part of the course is that of local bifurcation, in which a base state switches from being linearly stable to linearly unstable (or vice versa) as an externally applied control parameter is smoothly varied. Of course, this concept has already been encountered in part I of the course. The new focus here will be on the way in which nonlinear terms can (for example) restabilise the system, beyond the threshold of instability, in a new nonlinear state. In this section, we introduce the general theory of bifurcations in the context of some simple model equations.

Sections 8 and 9
After Easter, we will turn our attention to the so called Stuart-Landau equation and Ginzburg-Landau equation in Sections 8 and 9 respectively. Details to follow after the vacation...