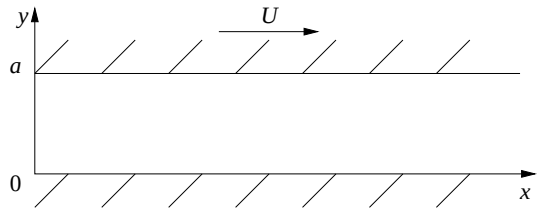
Laminar Boundary Layers

Problem sheet 1. Navier Stokes equations

1. Couette-Poiseuille flow

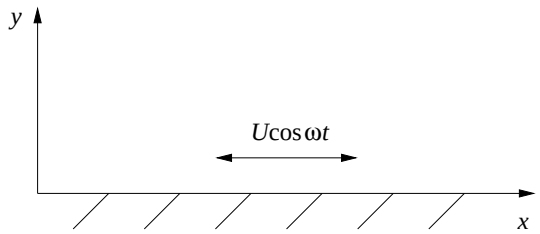
Consider incompressible flow between two horizontal plates. The plate at $y = 0$ is stationary, while the plate at $y = a$ is moving with constant velocity U. A constant pressure gradient is applied along the x direction.



a) Determine the steady velocity profile of the fluid.
b) Discuss the form of this profile with respect to the sign of the pressure gradient.
c) Calculate the rate of throughput, the average velocity and the drag on the plate y=0.

2. Shear flow near an oscillating plate

An incompressible fluid is in contact with a flat plate at $y = 0$, which is itself in harmonic motion with frequency ω.



a) Determine the motion induced in the fluid.
b) From the solution to part a), identify the distance over which the motion of the plate has an appreciable effect on the fluid (*e.g.* for water, which has a dynamic viscosity $\nu = \mu/\rho = 10^{-6}\mathrm{m}^2\mathrm{s}^{-1}$, assuming a frequency $\omega = 1$ MHz).
c) Calculate the drag force on the plate.

3. Show that

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_i}(\rho u_i E) = \rho \frac{DE}{Dt},$$

where ρ denotes the density of the fluid. Note that E in this expression can represent any function. For example, if it represents a velocity component u_j, then the left hand side represents the rate of change of momentum in the x_j direction, whereas the right hand side represents the mass $\times$ the acceleration in the x_j direction.

4. Stress tensor

Show that the stress tensor in the Cauchy equations is symmetric by taking moments about the axes through the centre of a cubic element of fluid.