

4.3 Laminar Boundary layer separation

We now discuss the phenomenon of boundary layer separation, in which the boundary layer peels away from the solid surface as the result of an adverse pressure gradient opposing the flow along it. This is commonly encountered in flows round the back of blunt obstacles: see Fig. 11 and recall the case $m < 0$ at the end of the previous section. We start in Sec. 4.3.1 by examining the conditions under which an adverse pressure gradient exists. We then show how this causes separation in Sec. 4.3.2. Finally, we discuss practical techniques for eliminating this undesirable phenomenon.

4.3.1 Adverse pressure gradient, $\partial p/\partial x > 0$

Consider the inviscid region (outside the BL) of flow past a blunt object such as the cylinder in Fig. 11. Due to the space restriction imposed by the object, the streamlines become more closely packed between the nose D and waist E , then space out again at the tail F . To maintain a constant throughput, therefore, the flow must speed up from D to E then slow down from E to F . Looking back at Euler's equations 33 to 35, we see that the only impetus for acceleration and deceleration in inviscid flow is the pressure term, ∇p . So there must exist a pressure gradient $\partial p_e/\partial x < 0$ to accelerate the fluid along the surface x from D to E and a gradient $\partial p_e/\partial x > 0$ to decelerate it again between E and F . (As usual, x is the curvilinear coordinate round the surface and y the local normal.) This is confirmed by Bernoulli's equation $\frac{1}{2}\rho u_e^2 + p_e = \text{constant}$ along a streamline. This inviscid part of the flow is symmetric: fluid arrives at F with the same velocity it had departing D . Potential energy (PE) in the pressure field is converted to kinetic energy between D and E , then fully back to PE between E and F , because there is no source of dissipation in an inviscid fluid.

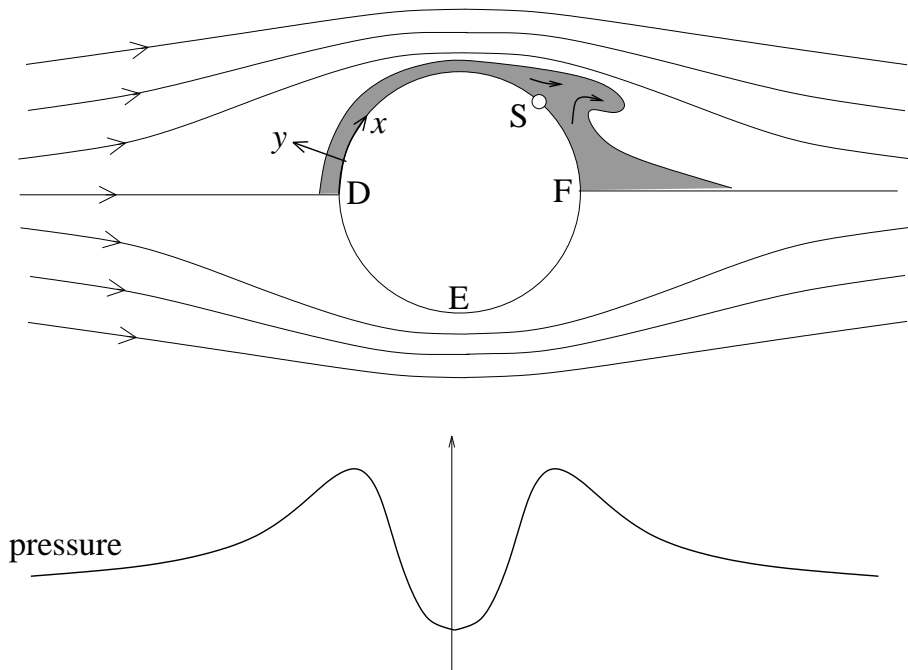


Figure 11: Pressure field and separation point for flow past a cylinder.

What about the pressure inside the boundary layer? Looking back at the derivation of the BL equations, Sec. 3, we recall that the pressure does not vary across the thickness of the layer: $\partial p/\partial y = 0$, Eqn. 48. Hence, the variation of the pressure with x along the inside the BL must be the same as that predicted by inviscid theory just outside it: $p(x) = p_e(x)$. (This is often expressed by saying that the pressure is “impressed” on the BL from the outside.)

4.3.2 Separation

We have seen that an adverse pressure gradient, $dp_e/dx > 0$, must exist round the back of a blunt obstacle. We have also commented that, in the inviscid region, the fluid has just enough impetus to surmount this pressure barrier and reach F with the same kinetic energy it had at D . Inside the BL, on the other hand, energy is dissipated by viscous drag: the fluid can therefore be left with insufficient energy to reach F so that it halts at some “separation” point S , beyond which it reverses and peels away from the surface, Fig. 11.

This intuitive picture is confirmed by examining the BL equation 45. At the surface itself, $y = 0$, we have BCs $u = v = 0$ so that

$$\frac{1}{\mu} \frac{dp_e}{dx} = \left(\frac{\partial^2 u}{\partial y^2} \right)_{y=0}. \quad (131)$$

At the body surface, therefore $\partial^2 u/\partial y^2$ must always have the same sign as the pressure gradient dp_e/dx . In contrast, we know that at the exterior edge of the BL we must have $\partial^2 u/\partial y^2 < 0$ in order for $\partial^2 u/\partial y^2 \rightarrow 0$ from below as the profile smoothly matches onto the exterior inviscid flow. (Look at Fig. 13 to convince yourself of this.) At an x -station with a favourable pressure gradient ($dp_e/dx < 0$), therefore, $\partial^2 u/\partial y^2 < 0$ for all y and the velocity profile has the form in Fig. 12, left. At an x -station with an adverse gradient ($dp_e/dx > 0$), on the other hand, it develops a point of inflexion at the crossover between the inner region of $\partial^2 u/\partial y^2 > 0$ and the outer region of $\partial^2 u/\partial y^2 < 0$ (Fig. 12, right). As one proceeds further along the boundary layer, Fig. 13, the profile can eventually become so distorted that we have

$$\left(\frac{\partial u}{\partial y} \right)_0 = 0 \quad (132)$$

at the surface, defining the point of separation. The flow pattern then appears as in Fig. 14. Beyond the separation point, the boundary layer becomes considerably thicker. Our boundary layer equations then cease to apply and any analysis becomes very challenging.

Once separation has occurred, the flow behind the separation point typically comprises a vortex filled wake that differs drastically from the predictions of inviscid theory. In particular, it exerts a suction drag on the body that is typically much larger than any viscous drag exerted by the boundary layer itself. In engineering applications, this is clearly undesirable. How can it be eliminated? The most obvious solution is to avoid the use of blunt bodies: separation is minimised by the careful tapering of aerofoils, for example. Other techniques include applying boundary layer suction at a porous surface; and (more difficult) rotating the surface with the flow.

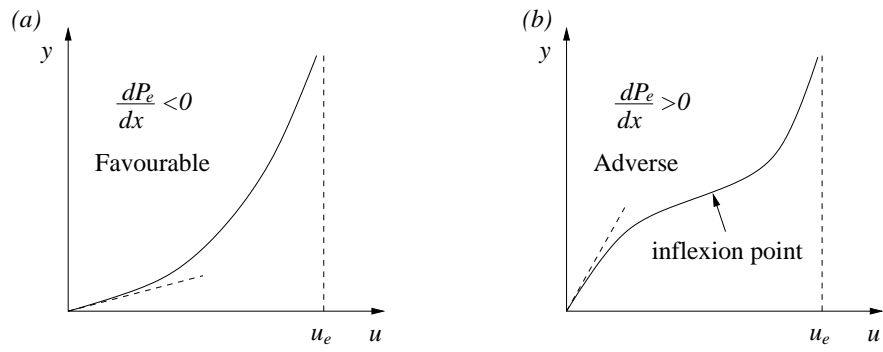


Figure 12: Sketch of the velocity profile in the boundary under (a) a favourable pressure gradient, (b) an adverse pressure gradient.

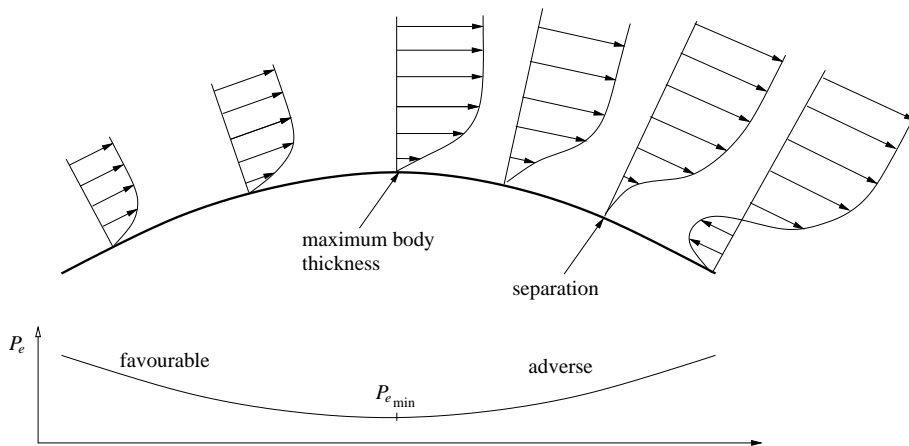


Figure 13: Boundary layer separation over a body

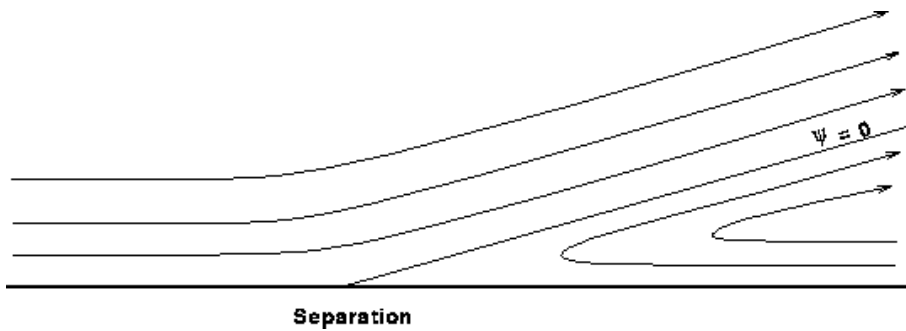


Figure 14: Flow pattern near the point of separation.