Hydrodynamic Stability. Part I: Linear stability analysis

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Abstract

The equations of fluid dynamics admit some simple laminar flow states as stationary solutions. In some cases these laminar states become unstable, leading to more complicated (patterned/turbulent) states. Part I of the course concerns the initial onset of such instabilities, studied via the dynamics of tiny perturbations to the laminar state. If such perturbations grow in time, the laminar state is said to be linearly unstable.

Section 1: Introduction
We start with a recap of the basic (Navier Stokes) equations of fluid dynamics. We then introduce the concept of linear instability, and outline the basic procedure involved in a linear stability analysis. This procedure will be followed in each subsequent section 2-4 below, and can be referred back to as the template used in each case.

Section 2: Rayleigh-Bénard convective instability
When a horizontal layer of a fluid is heated from below, a temperature gradient is established. The cooler fluid near the top of the layer is then denser than the warmer fluid underneath it. For a large enough temperature gradient this can buoyancy effect can cause an overturning instability, leading to convection rolls.

Section 3: Centrifugal instability
Flows with curved streamlines can be unstable due to the centrifugal effects of rotation. Here we consider centrifugal instabilities in inviscid fluids. Our main focus is Rayleigh's criterion for the instability of a basic swirling flow with an arbitrary dependence of angular velocity on the distance from the axis of rotation. We also discuss an analogy between these curvature driven instabilities and the thermal instabilities of Sec. 2.

Section 4: Shear flow instability
Finally we consider the linear stability of two-dimensional or axisymmetric flows with parallel streamlines. We derive the famous Orr-Somerfeld equation governing the stability of uni-directional shear flows, for viscous fluids. We then turn to inviscid fluids. Here we discuss Rayleigh's inflexion point theorem, which states that a necessary condition for inviscid instability is the presence of an inflexion point in the initial laminar flow profile.